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Nonlinear Dynamic Behavior of Plastics

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Synopsis

The dynamic behavior at low frequencies of Penton (chlorinated polyether) which displays nonlinear response is described in terms of a free energy function and a dissipated energy function. The variation of the energy functions with frequency and magnitude of stress is determined experimentally from hysteresis tests. The dynamic properties of the material are expressed in terms of storage and loss modulus functions. In addition the dynamic behavior of the material at low frequencies is predicted from uniaxial static creep properties, using a modified superposition principle.

INTRODUCTION

In recent years there has been a great increase in the use of plastics as structural materials. This has increased the need for understanding the mechanical behavior of plastics under static and dynamic loading conditions. Most plastics display combined elastic and viscous properties and, therefore, require experiments in which either stress or strain is varied as functions of time and the strain or stress is observed. For a plastic material which has linear relationship between stress and strain, the static and dynamic properties can be specified, in principle, by time-dependent functions such as creep and relaxation functions obtained from static tests and complex modulus function obtained from dynamic tests.[1] It is relatively difficult, however, to characterize a material having a nonlinear response. In this paper, dynamic behavior of a plastics material that displays nonlinear response has been described in terms of a free energy function that represents the amount of energy stored and a dissipated energy function that represents the amount of energy dissipated during deformation in the material. The variation of these energy functions with frequency and level of stress has been determined experimentally. The dynamic properties of the nonlinear plastics material have been expressed in terms of a storage modulus and a loss modulus. Just as in the case of linear materials, it is found that the loss and storage modulus are functions of frequency alone but their variation with frequency is found to correspond to the usual variation noted in the case of a linear viscoelastic material.[2,3] Finally, the dynamic behavior at low frequencies has been predicted from static creep properties of the material.

EXPERIMENTAL

Material

The material used in this investigation was a chlorinated polyether, designated by the tradename, Penton. This thermoplastic which is used extensively as a structural material has a high molecular weight of 250,000–350,000. The Penton monomer, a reaction product of pentaerythritol, is a chlorinated oxetane, 3,3-bis(chloromethyl)oxetane.

The specimens used were injection-molded. These specimens were supplied by Hercules Powder Company, Wilmington, Delaware. A flat specimen, shown in Figure 1*a*, was used in the uniaxial creep tests, and a round specimen, shown in Figure 1*b*, was used in the dynamic tests.

Experimental Arrangement

A conventional lever-arm type creep machine was used for conducting creep experiments. A Tinius Olsen 12,000-lb. capacity screw-driven machine was used for the dynamic tests. The machine is equipped with an automatic cycling device that enables one to impose triangular load history on the specimen. The rate of loading on the specimen may be varied by adjusting the rate of the crosshead movement. The crosshead speed may be varied from 0.02 to 20.00 in./min. The strains were measured by a clip gage and were recorded in a X–Y recorder through a strain gage convector.

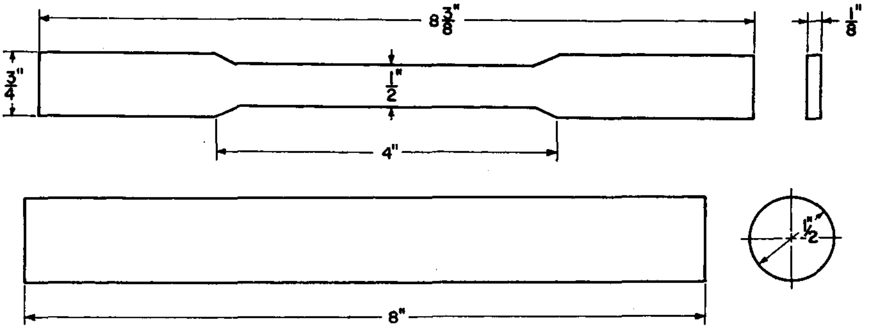

Fig. 1. Specimens used: (*a*) flat type Penton specimen used in creep tests; (*b*) round type Penton specimen used in dynamic tests.

Experimental Program

The experimental program consisted of two parts. The first one was connected with obtaining static properties for the time range used in dynamic tests. The second part included dynamic tests in the range of 0.0848–14.3 cycles/min. Static properties of the material were evaluated by conducting uniaxial tension creep tests at various stress values in the range 1980–4000 psi. The creep was conducted for a duration of 30 min. only. The dynamic properties were evaluated by subjecting solid cylin-

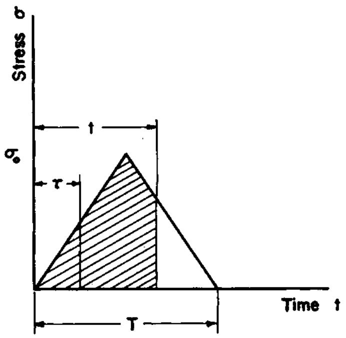

Fig. 2. The stress pattern in dynamic test.

drical specimens to a triangular tension stress history as shown in Figure 2. During any single test the maximum stress level was kept constant. The time for one cycle or the frequency of fluctuation was measured and was found to be constant after the application of the first few cycles on the specimen. The hysteresis loops were automatically recorded and from the chart the free energy and the dissipated energy were determined.

EXPERIMENTAL RESULTS AND THEORETICAL INTERPRETATION

In Figure 3 are shown the test results in uniaxial creep up to a stress level of 4000 psi and up to a duration of 30 min. The nonlinear behavior of the material in uniaxial creep is clearly illustrated in Figure 4, where the ratio of creep strain to stress, ϵ/σ is shown plotted against stress for various fixed values of time. For a material with linear response the ratio (ϵ/σ) for any fixed value of time remains constant with stress. Double logarithmic plots of creep strain versus uniaxial stress indicated that creep behavior of the material in uniaxial tension can be reasonably expressed by the empirical relation, eq. (1):

$$\epsilon = J(t)\sigma^n \tag{1}$$

where $J(t)$ is a creep compliance function (a function of time alone) and n is a material constant. The creep compliance function is found to obey the relation

$$J(t) = J_0 t^m \tag{2}$$

where J and m are material constants. The values of the constants are $J_0 = 1.12 \times 10^{-7}$, $n = 1.539$, and $m = 0.0743$.

Figure 5 shows the experimentally obtained free energy and dissipated energy for stress levels of 2000, 3000, and 4000 psi plotted in terms of logarithm of frequency of fluctuation. It was found that the total energy can be expressed by

$$W = W_1 + W_2 = E_1(f)\sigma^p + E_2(f)\sigma^q \tag{3}$$

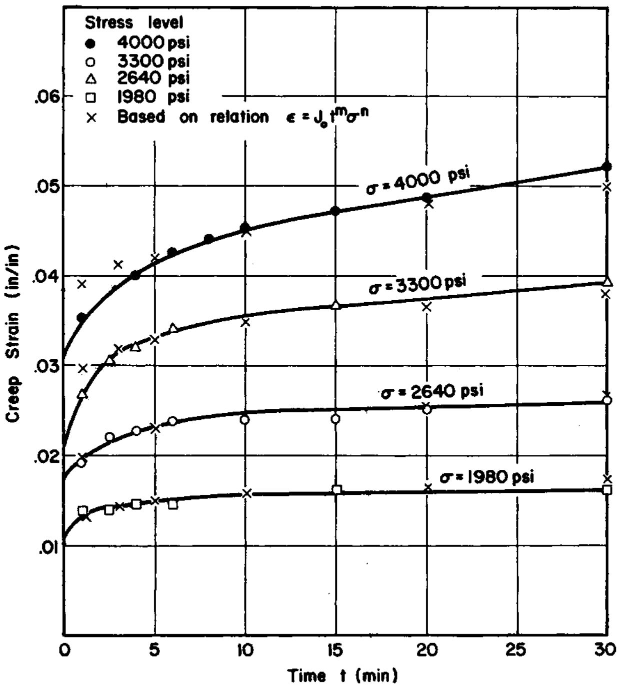

Fig. 3. Creep strain vs. time curves for Penton.

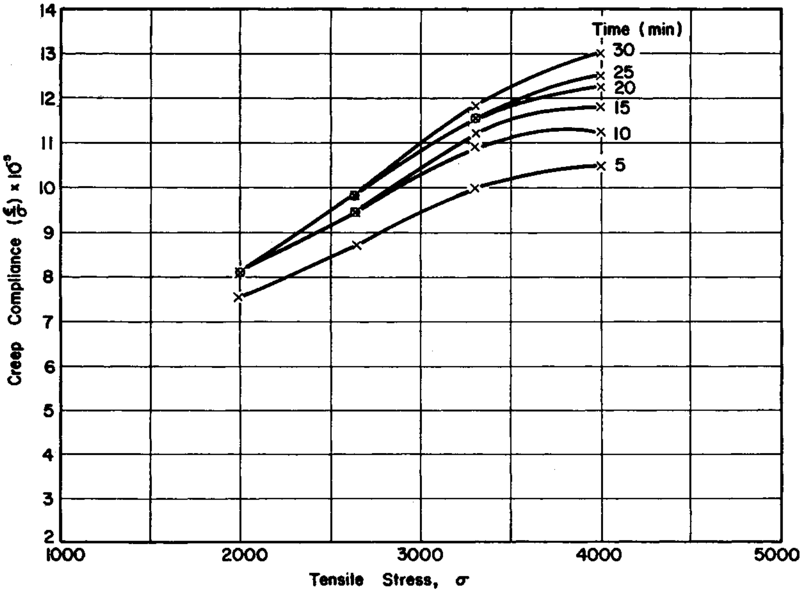

Fig. 4. Variation of creep compliance with stress for Penton.

where W is the total energy in the system, W_1 is the free energy, $E_1(f)$ is the storage modulus function (function of frequency), $E_2(f)$ is the loss modulus function (function of frequency), and p, q are material constants. The material constants for the material were found to be $p = 2.6$ and $q = 1.93$.

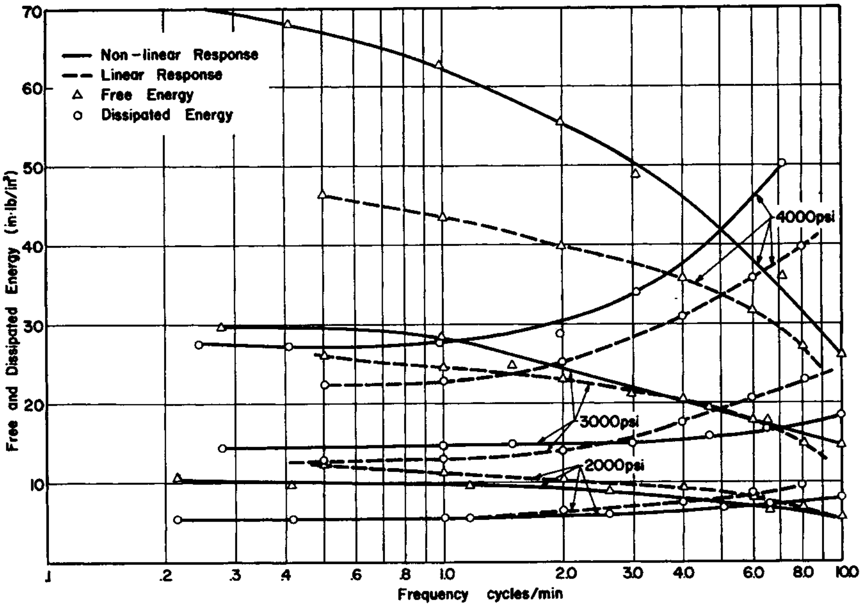

Fig. 5. Free energy and dissipated energy vs. log frequency in tension for Penton.

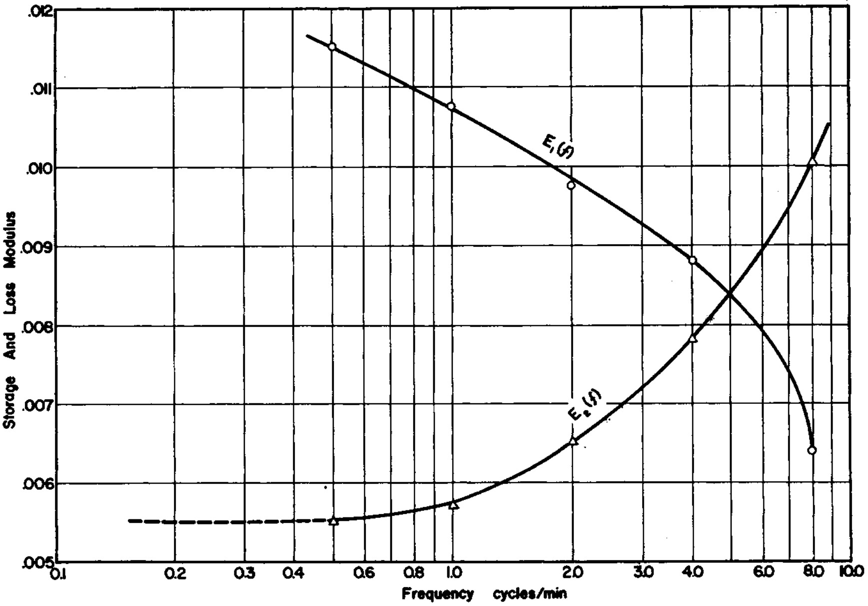

Fig. 6. Storage and loss modulus vs. frequency for Penton.

The variation of storage modulus and loss modulus with frequency is shown in Figure 6.

Predictions of Dynamic Creep Strains

Consider a material for which the creep curves for different uniaxial stress levels are geometrically similar. Then the creep strain for such a material can be expressed by the relation:

$$\epsilon = J(t)F(\sigma) \tag{4}$$

where $F(\sigma)$ is a stress function (function of stress alone) and $J(t)$ is the creep compliance function (function of time alone). For a linear material $F(\sigma)$ becomes just equal to σ, but for a nonlinear material $F(\sigma)$ can be any function of stress depending upon the type of the material.

For a material that has linear response it is only enough to specify the material properties by a creep compliance function, and the linearity between stress and strain allows one to describe the behavior of the material under any type of load pattern by the Boltzmann superposition principle.[4] For a nonlinear material, however, the Boltzmann superposition principle in the form it was originally suggested does not apply. It is quite reasonable to replace it by means of a modified superposition principle[5] as shown in eq. (5):

$$\epsilon(t) = \int_0^t J(t - \tau)[dF(\sigma)/d\tau]d\tau \tag{5}$$

where $\epsilon(t)$ is creep strain at time t and τ is the time at which a certain increment of load took place.

By eq. (5), the strain as a function of time can be determined for the given load history as shown by Figure 2. The load history represented by Figure 2 can be expressed by the relations:

$$\begin{aligned} \sigma &= 0 && t < 0 \\ \sigma &= (2\sigma_0/T)t && 0 < t < T/2 \\ \sigma &= 2\sigma_0(1/2 - t/T) && T/2 < t < T \end{aligned} \tag{6}$$

where T is the time for one cycle and σ_0 is the amplitude of stress.

On substituting the stress function $F(\sigma)$ and the creep compliance function $J(t)$ from eqs. (1) and (2), respectively, in eq. (5), the expressions for the strain component at time t for the stress history given in eqs. (6) are

$$\epsilon(t) = \int_0^t J_0(t - \tau)^m \frac{d}{d\tau}\left(\frac{2\sigma_0}{T}\tau\right)^n d\tau \quad 0 < \tau < t \tag{7}$$

$$\begin{aligned} \epsilon(t) = &\int_0^{T/2} J(t - \tau)^m \frac{d}{d\tau}\left(\frac{2\sigma_0}{T}\tau\right)^n d\tau \\ &+ \int_{T/2}^t J_0(t - \tau)^m \frac{d}{d\tau}\left[2\sigma_0\left(1/2 - \tau/T\right)\right]^n d\tau \quad T/2 < \tau < t \end{aligned} \tag{8}$$

From eq. (7) and the second of eqs. (6), the total energy per cycle can be calculated:

$$W = \int_0^{T/2} (2\sigma_0/T)t d\epsilon(t) \quad 0 < t < T/2 \tag{9}$$

Similarly, from eq. (8) and the third of eqs. (6) the free energy (the energy stored) in the material per cycle is

$$W_1 = \int_{T/2}^{T} 2\sigma_0(1/2 - t/T)d\epsilon(t) \quad T/2 < t < T \tag{10}$$

From eqs. (9) and (10), the dissipated energy per cycle can be determined as

$$W_2 = (W - W_1) \tag{11}$$

In Table I are compared the stored and dissipated energies calculated from eqs. (9), (10), and (11) and the corresponding experimental values.

TABLE I
Comparison of Theoretical and Experimental Energies

Stress level, psi	Frequency, cycles/min.	Total energy, in.-lb./in.^{3}	Free energy, in.-lb./in.^{3}		Dissipated energy, in.-lb./in.^{3}	
			Theoretical	Experimental	Theoretical	Experimental
2000	4	10.874	10.252	8.00	0.622	6.2
	2	11.465	10.857	9.00	0.608	5.95
	1	11.833	11.262	9.99	0.571	5.5
	1/2	12.301	11.651	10.00	0.650	5.5
3000	4	15.008	14.366	20.0	0.642	15.5
	2	15.287	14.469	24.2	0.818	14.9
	1	16.443	15.4755	28	0.9675	14.8
	1/2	17.361	16.4	29.5	0.961	14.5
4000	4	15.102	14.248	46	0.854	37.5
	2	15.936	15.404	55	0.532	30.5
	1	17.076	16.176	62	0.90	27.5
	1/2	18.112	17.036	67	1.076	22.5

DISCUSSION

The experimental results in uniaxial static creep (Fig. 4) and dynamic behavior as represented by eq. (3) indicate the material behavior is nonlinear. In Figure 5 is shown the variation of free and dissipated energies if the material response was linear. These energies were calculated by assuming the exponents in eq. (3) as 2. It can be seen from Figure 5 that as the level of stress is decreased, the energies for nonlinear and linear cases tend to coincide for the frequency range considered. Figure 5 also shows that the free energy decrease with frequency is more gradual in the case of linear response than in the case of nonlinear response. Table I shows that the prediction of free energy from uniaxial creep behavior is very good for the stress level of 2000 psi. The deviation between experimental and theoretical values becomes greater as the stress level is increased. The predicted dissipated energy values do not compare well with experimental results. It appears the experimental dissipated energy values are greater than the theoretical values by one order of magnitude.

CONCLUSIONS

The dynamic properties at low frequencies of nonlinear materials such as chlorinated polyether can be described by storage and loss modulus functions just as in the case of linear viscoelastic materials. The storage and loss modulus functions may be obtained from the estimation of energies stored and dissipated in hysteresis experiments. The prediction of dynamic properties at low frequencies from uniaxial static creep behavior by use of the modified superposition principle did not yield good results.

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Résumé

Le comportement dynamique à basses fréquences du penton (polyéther chloré) qui présente une réponse non-linéaire, est décrit sous forme de fonction d'énergie libre et de fonction d'énergie dissipée. La variation des fonctions d'énergie avec la fréquence et la grandeur de la tension est déterminée expérimentalement par des tests d'hystérésis. Les propriétés dynamiques de la substance sont exprimées en termes de fonctions de conservation et de module de perte. En outre le comportement dynamique de la substance à basse fréquence est prédit à partir des propriétés uniaxiales du fluage statique en employant un principe de superposition modifié.

Zusammenfassung

Das dynamische Verhalten von Penton (chlorierter Polyäther) bei niedrigen Frequenzen, das eine nichtlineare Abhängigkeit zeigt, wird als Funktion der freien und der dissipierten Energie beschrieben. Die Abhängigkeit der Energiefunktionen von der Frequenz und der Spannungsgrösse wird experimentell aus Hystereseuntersuchungen bestimmt. Die dynamischen Eigenschaften des Polymeren werden als Funktionen des Real- und Imaginärteils des Moduls ausgedrückt. Überdies wird das dynamische Verhalten des Materials bei niedriger Frequenz aus den einaxialen statischen Kriecheigenschaften unter Verwendung eines modifizierten Superpositionsprinzips vorhergesagt.

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